

# A Laplace-like formula for the energy dependence of the nuclear level density parameter

B. Canbula<sup>a,\*</sup>, R. Bulur<sup>a</sup>, D. Canbula<sup>a</sup>, H. Babacan<sup>a</sup>

<sup>a</sup>*Department of Physics, Faculty of Arts and Sciences,  
Celal Bayar University, 45140, Muradiye, Manisa, Turkey*

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## Abstract

Collective effects in the level density are not well understood, and including these effects as enhancement factors to the level density does not produce sufficiently consistent predictions of observables. Therefore, collective effects are investigated in the level density parameter instead of treating them as a final factor in the level density. A new Laplace-like formula is proposed for the energy dependence of the level density parameter, including collective effects. A significant improvement has been achieved in agreement between observed and predicted energy levels. This new model can also be used in both structure and reaction calculations of the nuclei far from stability, especially near the drip lines.

*Keywords:* nuclear level density, semi-classical, collective motion, harmonic oscillator, Coulomb potential, rotational modes, vibrational modes

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## 1. Introduction

Nuclear level density (NLD), which is the number of the excited levels around an excitation energy, has been studied for nearly eight decades. The knowledge about the NLD is the key of the accurate Hauser-Feshbach calculations for the compound-nucleus cross sections. It becomes an obligation to use the level density function in the case of incompleteness of the experimental information on the energy levels, or at high excitation energies, which levels become very narrowly spaced, or even continuous. The first study on this subject was proposed by Bethe [1], who introduced the Fermi gas model, and many authors have been studied on this subject extensively with several methods [2–12]. Although it is highly desired to use microscopic models, phenomenological models are still useful and popular due to their simplicity and ease of application. On the other hand, these models usually have several free parameters to be adjusted the experimental data, namely the mean resonance spacings and the discrete level schemes. The fitting and the subsequent extrapolation of the parameters are the main limitations to use these models reliably for the nuclei far from stability. After the pioneering work of Tanihata [13], the nuclei near the drip lines have been subject of interest because of their unusual

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\*Corresponding author

*Email address:* bora.canbula@cbu.edu.tr (B. Canbula)

properties. Therefore, to propose a level density model, which can be used as a reliable tool for the theoretical calculations of the reactions involving light exotic beams, is an outstanding problem in nuclear structure.

The other crucial problem of the level density is the collective enhancement. The coherent collective nuclear excitations cause an increase in the level density and play a dominant role at the low-energy region before damping with the increasing excitation energy. Therefore, without taking into account these effects, it is impossible to describe the first few low-lying excited states of the nucleus. The collective effects can be separated into two parts, namely vibrational and rotational. While the rotational excitations make contributions to the level density only for deformed shapes, the vibrational effects should be considered even for the spherical nuclei. In spite of many studies on the collective effects in the level density [14–16], the results are not at the level of expectation. Hence, this problem still remains unsolved and requires further investigation.

In the light of the above discussion, the objective of this paper is to propose a new method to include the collective effects into the level density formalism and improve the usability of the level density in the reaction calculations of the exotic nuclei. This paper is organized as follows: A brief introduction of the nuclear level density is given in Section 2. The results of the calculations are presented in Section 3. Finally, in Section 4, we summarize our results and discuss their significance.

## 2. Theory

According to the Fermi gas model, nucleus treated as a system of non-interacting nucleons and collective levels are absent, therefore excited levels arise only from the single-particle states with equally spaced. Under these assumptions, the level density of a double fermion system, which is formed from protons and neutrons, is given as a function of effective energy  $U = E_x - \Delta$ , level density parameter  $a$ , spin  $J$ , spin cut-off parameter  $\sigma^2$ , with equiparity distribution [1, 17]

$$\rho(U, J, \Pi) = \frac{1}{2} \frac{2J+1}{2\sqrt{2\pi}\sigma^3} \exp\left[-\frac{(J+\frac{1}{2})^2}{2\sigma^2}\right] \frac{\sqrt{\pi} \exp[2\sqrt{aU}]}{12 a^{1/4} U^{5/4}} \quad (1)$$

Spin cut-off parameter can be written as  $\sigma^2 = TI/\hbar^2$  in the simplest form where  $T$  is the nuclear temperature, and  $I$  is the moment of inertia. The energy shift  $\Delta = \delta + n \frac{12}{\sqrt{A}}$  where  $n$  is  $-1$  for odd-odd,  $1$  for even-even,  $0$  for odd nuclei and  $\delta$  remains as an adjustable parameter to fit. The total level density can be obtained by summing (1) over all spins

$$\rho^{\text{tot}}(U) = \frac{1}{12\sqrt{2}\sigma} \frac{\exp[2\sqrt{aU}]}{a^{1/4} U^{5/4}}. \quad (2)$$

This equation provides a simple and successful description of the level density, especially around the neutron separation energies, but it also causes a divergence problem when excitation energy goes to zero. This problem remained unsolved until 1985 [18], and Demetriou [3] proposed a convenient solution, which is also used in this study, in 2001.

The main variable of the NLD is the level density parameter  $a$  and commonly given by Ignatyuk's [19] formula depending on the excitation energy as given below

$$a(U) = \tilde{a} \left( 1 + \delta W \frac{1 - \exp[-\gamma U]}{U} \right). \quad (3)$$

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$\delta W$  is the microscopic correction term of the liquid drop mass formula and  $\tilde{a}$ , the asymptotic level density parameter, is the limit value of  $a$  that is reached at high excitation energies, especially beyond the neutron separation energy. The damping parameter  $\gamma$  is given as  $\gamma = \gamma_1/A^{1/3}$  where  $\gamma_1$  is an adjustable parameter that determines how rapidly  $a$  goes to  $\tilde{a}$  and the direction of this damping depends on the sign of the  $\delta W$ .

In the early studies of the level density, the level density parameter was taken to consist of only its asymptotic value, therefore, it was independent of the excitation energy. This parameter is usually deduced from the experimental data by using a parameterized equation [20–22] as well as it can be calculated theoretically from the proton and neutron single-particle level densities at corresponding Fermi energies

$$\tilde{a} = \frac{\pi^2}{6} [g_p(E_F^p) + g_n(E_F^n)]. \quad (4)$$

One can use the semi-classical approximation to calculate the single-particle level density at a single-particle energy  $\varepsilon$  with the spin degeneracy [23, 24]

$$g(\varepsilon) = \frac{2}{\pi} \left( \frac{2m}{\hbar^2} \right)^{3/2} \int r^2 \sqrt{\varepsilon - V(r)} dr \quad (5)$$

where  $V(r)$  and  $m$  are the average simple potential and the mass of the nucleon, respectively. The value of the Fermi energy can be found from the following conservation condition between the nucleon number  $\mathcal{N}_\alpha$  and the single-particle level density  $g_\alpha$

$$\mathcal{N}_\alpha = \int_{-\infty}^{E_F^\alpha} g_\alpha(E) dE, \quad \mathcal{N}_\alpha = \{N, Z\}. \quad (6)$$

With the model described so far it is possible to calculate two observables, mean resonance spacings

$$\frac{1}{D_0^{\text{theo}}} = \sum_{J=|I-\frac{1}{2}|}^{J=I+\frac{1}{2}} \rho(S_n, J, \Pi) \quad (7)$$

and cumulative levels up to an excitation energy  $E_x$  from the lower-level  $N_L$  with the energy  $E_L$ ,

$$N_{\text{cum}}(E_x) = N_L + \int_{E_L}^{E_x} \rho^{\text{tot}}(E_x) dE_x \quad (8)$$

Table 1: Goodness-of-fit estimators of the existing phenomenological level density models.

Model	Type	$f_{\text{rms}}$	$f_{\text{lev}}$
BSFGM [20]	Effective	1.68	28.5
BSFGM [20]	Collective	1.71	35.3
CGCM [20]	Effective	1.76	24.2
CGCM [20]	Collective	1.77	47.8
GSM [20]	Effective	1.78	28.0
GSM [20]	Collective	1.94	47.4

with the adjustable parameters  $\delta$  and  $\gamma_1$ . For  $N$  nuclei, the quality of these calculations is given by the rms deviation factor for mean resonance spacings

$$f_{\text{rms}} = \exp \left[ \frac{1}{N} \sum_{i=1}^N \left( \ln \frac{D_{0,i}^{\text{theo}}}{D_{0,i}^{\text{exp}}} \right)^2 \right]^{1/2} \quad (9)$$

and the average goodness-of-fit estimator for discrete levels

$$f_{\text{lev}} = \frac{1}{N} \sum_{i=1}^N \sum_{k=N_L^i}^{N_U^i} \frac{[N_{\text{cum}}^i(E_k) - k]^2}{k} \quad (10)$$

from a lower-level  $N_L$  with the energy  $E_L$  to an upper-level  $N_U$  with the energy  $E_U$  [25].

Table 1 shows the agreement between the experimental data and predicted observables from the phenomenological models, for which each model has two different types according to their ways of handling the low-lying collective levels. Effective models try to describe all excited levels, including the collective ones by fitting the adjustable parameters to experimental data. In contrast to this approach, collective models consider the total level density given by Eq. (2) as an intrinsic level density, which describes only pure single particle excitations, and use enhancement factors that explicitly account the collective effects

$$\rho(U) = K_{\text{rot}} K_{\text{vib}} \rho_{\text{int}}(U) \quad (11)$$

where  $K_{\text{rot}}$  and  $K_{\text{vib}}$  are the coefficients for the rotational and vibrational enhancement respectively. Even if it seems more physical to use collective models for theoretical calculations, the goodness-of-fit estimators given in Table 1 show that significantly better agreement with experimental data is achieved when effective models are used.

This situation motivated us to search for a different method that can deal with collective effects and produce more accurate predictions than existing phenomenological models. To use enhancement factors for the level density expression is an obviously deficient and delayed attempt for describing the collective levels, therefore these effects must be included to level density calculations from the beginning. The most suitable candidate to include these effects seems to be the level density parameter. Since we know very little about the collective effects, it will be reasonable to start with considering the level density parameter as a single free parameter in the model and fit it to experimental data. The obtained results for  $^{146}\text{Nd}$  is shown in Figure 1. Here  $^{146}\text{Nd}$  is randomly chosen because most of the nuclei exhibit similar behavior.

Two different striking results can be deduced from Figure 1. First, with the increasing excitation energy, the level density parameter goes to a different limit from the asymptotic level density parameter given by Eq. (4). The main reason of this difference is that the single-particle levels bunch together around the Fermi energy [27] which we have neglected in our calculations. This difference corresponds to the microscopic (or the shell) correction energy, so we must fix the calculated value of the Fermi energy from Eq. (6). We may also consider a further modification to the Fermi energy for the pairing effects. With this modification, all excitation energies have been corrected by an amount of  $\Delta$  and therefore, all excitation energies have been transformed into effective excitation energies. With these modifications, the corrected Fermi energy is given by

$$E_F^* = E_F + S(N, Z) - \Delta \quad (12)$$

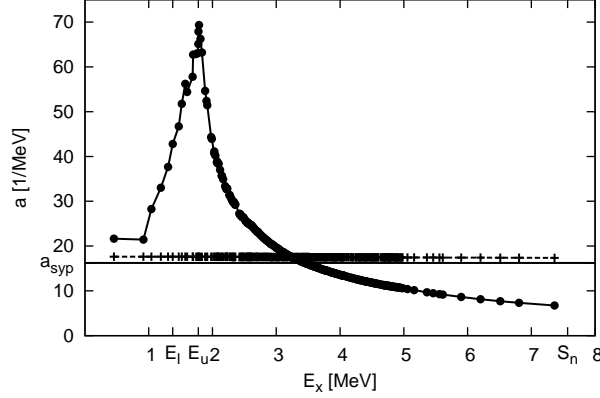


Figure 1: The results of level density parameter fit for  $^{146}\text{Nd}$ . Fitted values (close circles) are compared to calculated values (plus signs) from Ignatyuk's formula (3) and calculated asymptotic level density parameter (4) (solid line).

where  $S(N, Z)$  is the shell correction energy of the liquid drop model [27]. We did not need to use the adjustable parameter  $\delta$ , so the pairing correction is just given as  $\Delta = n \frac{12}{\sqrt{A}}$  with  $n$  is  $-1$  for odd-odd,  $1$  for even-even,  $0$  for odd nuclei. This way of handling the pairing correction also enables to make calculations with the excitation energies  $E_x \leq \Delta$ . Consequently, the corrected asymptotic level density parameter is given by the equation below:

$$\tilde{a} = \frac{\pi^2}{6} [g_p(E_F^{p*}) + g_n(E_F^{n*})]. \quad (13)$$

For the excitation energies higher than the neutron separation energy, the level density parameter is approximately equal to the asymptotic level density parameter. Thus, the correct description of the asymptotic level density parameter is the only way to improve the accuracy of the model in this region.

The second and even more important result coming out from Figure 1 is the peak around 2 MeV. It is well known that the origin of this extraordinary behavior at this energy is collective motion of the nucleons, in other words; this energy corresponds to the first phonon state arises from the vibrational motion [28, 29]. Furthermore, the energy dependence of the level density parameter appears as a Laplacian distribution rather than the exponential decay given by Ignatyuk's formula (3). All these unexpected results clearly show that it is an appropriate method to search the collective effects in the level density parameter. Therefore, we propose a new formula for the level density parameter

$$a(U) = \tilde{a} \left( 1 + A_c \frac{S_n}{U} \frac{\exp(-|U - E_0|/\sigma_f^3)}{\sigma_f^3} \right). \quad (14)$$

In above equation, the excitation energy is denoted by  $U$ , but it has same value as  $E_x$  because using Eq. (13) all excitation energies are transformed into effective ones, so this procedure may be called as the indirect back-shifting. The location of the peak can be described by the excitation energy of the first  $2^+$  state of even-even nuclei and

approximated as [29–31]

$$E_0 = 0.2\hbar\omega \quad (15)$$

where  $\hbar\omega = 41/A^{1/3}$  MeV. The Laplace distribution is desired to spread over an excitation energy range from the ground state to neutron separation energy at least and then both shell and collective effects should damp with the higher energies. Therefore, the scale parameter of the Laplace distribution must be related to neutron separation energy. To achieve this aim, we define a critical nuclear temperature as

$$T_c = \sqrt{\frac{S_n}{\tilde{a}}} \quad (16)$$

and we use the corresponding spin cut-off parameter at this temperature as the scale parameter

$$\sigma_c^2 = \frac{T_c I}{\hbar^2}. \quad (17)$$

to obtain the scale parameter  $\sigma_c^3 = \sigma_c^3/\tilde{a}$ . Here we use the perpendicular moment of inertia instead of the spherical one  $I_0 = 0.4MR^2$ , and it is given in terms of deformation parameters [32]

$$I = I_0 \left[ 1 + \sqrt{\frac{5\pi}{16}}\beta_2 + \frac{45}{28\pi}\beta_2^2 + \frac{15}{7\sqrt{5}\pi}\beta_2\beta_4 \right] \quad (18)$$

and provides rotational enhancement for deformed nuclei.

The last ingredient of Eq. (14) is  $A_c$ . We define this parameter as the collective amplitude, and it is closely related to the shell structure and the surface oscillations just like the low-frequency collective modes. Therefore,  $A_c$  should include the shell correction energy, but as the scale (spin cut-off) parameter, it must be at the same critical temperature (16). The temperature dependence of the shell correction energy is given by [23, 31, 33]

$$S(N, Z, T) = S(N, Z) \frac{\tau}{\sinh \tau} \quad (19)$$

where  $\tau = 2\pi^2 T/\hbar\omega$ . It would be useful to remind that we denote the shell correction with  $S(N, Z)$ , which equals to  $\delta W$  in Eq. (3), to establish the notation. It is correct but insufficient to use the shell correction energy as collective amplitude. It is crucial to take into account the surface oscillations to describe the collective excitations. The shape dependent shell correction energy can be written as [27]

$$S(N, Z, \text{Shape}) = M_{\text{exp}} - M_{\text{LDM}} \quad (20)$$

where  $M_{\text{LDM}}$  is the mass, which takes into account the small spheroidal distortions with the equation below:

$$M_{\text{LDM}} = M_0 + E\theta^2. \quad (21)$$

$E$  is a coefficient related to the fissility parameter  $x$  as  $E = (2/5)c_2 A^{2/3}(1-x)\alpha_0^2$  where  $\alpha_0^2 = 5(a/r_0)^2 A^{-2/3}$ .  $\theta = \alpha/\alpha_0$  is the deformation magnitude in terms of the deformation variable  $\beta^2$  where  $\alpha^2 = (5/4\pi)\beta^2$ . For further details see Ref. [27].  $M_0$  is the mass of the corresponding spherical nucleus and defined by the well-known formula of the finite-range liquid-drop model [27]

$$M_0 = M_N N + M_H Z + E_V + E_S + E_C \pm \frac{11}{\sqrt{A}} \quad (22)$$

where the volume energy  $E_V = -c_1 A$ , the surface energy  $E_S = c_2 A^{2/3}$ , the Coulomb energy  $E_C = c_3 \frac{Z^2}{A^{1/3}} - c_4 \frac{Z^2}{A}$ , the last term is negative for odd-odd, positive for even-even, and equals to zero for odd nuclei. Finally, the collective amplitude  $A_c$  is defined as the shape dependent shell correction energy at the critical temperature

$$\begin{aligned} A_c &= S(N, Z, T_c, \text{Shape}) \\ &= [M_{\text{exp}} - M_{\text{LDM}}] \frac{\tau_c}{\sinh \tau_c} \\ &= [M_{\text{exp}} - (M_0 + E\theta^2)] \frac{\tau_c}{\sinh \tau_c} \end{aligned} \quad (23)$$

where  $\tau_c = 2\pi^2 T_c / \hbar\omega$ .

### 3. Results and Discussion

With the level density model described so far, both global and local calculations can be made. In the global calculation, the asymptotic level density parameter must be obtained analytically by using Eq. (13) with the global potential parameters. In the present paper, we define  $V(r)$  as the sum of central, harmonic oscillator and the Coulomb potential terms:

$$V(r) = \frac{\hbar^2}{2mr^2} l(l+1) + V_{\text{HO}}(r) + V_C(r). \quad (24)$$

Harmonic oscillator potential is given by

$$V_{\text{HO}}(r) = \frac{1}{2} m\omega^2 r^2 \quad (25)$$

where  $\hbar\omega = 41/A^{1/3}$  MeV. The coulomb potential of the uniformly charged sphere is

$$V_C(r) = \begin{cases} \frac{Ze^2}{2R_C} \left( 3 - \frac{r^2}{R_C^2} \right) & r \leq R_C \\ \frac{Ze^2}{r} & r \geq R_C \end{cases} \quad (26)$$

and charge radius  $R_C$  is given by a simple formula  $R_C = 1.169 A^{0.291}$  which is obtained from a recent fit [34] to the latest nuclear charge radii data [35]. In the local calculation, the asymptotic level density parameter is adjusted to the experimental data for each nucleus separately.

Obtaining the asymptotic level density parameter by Eq. (13), including the shell and the pairing corrections, which is one of the novelties of this paper, has a considerable importance in view of the wide energy range above the neutron separation energy. For reaction calculations in this regime, the definition of the asymptotic level density parameter is almost the only way to improve the level density description. The ratio of the predicted mean resonance spacings from the global and local calculations to the experimental data are plotted in Figure 2. This ratio is the only indicator of the success of the level density models in the neutron separation energy regime. This ratio is in the range from 0.8 to 2 for the most of the nuclei, and this result is highly satisfactory compared to other phenomenological level density models. More importantly, there is

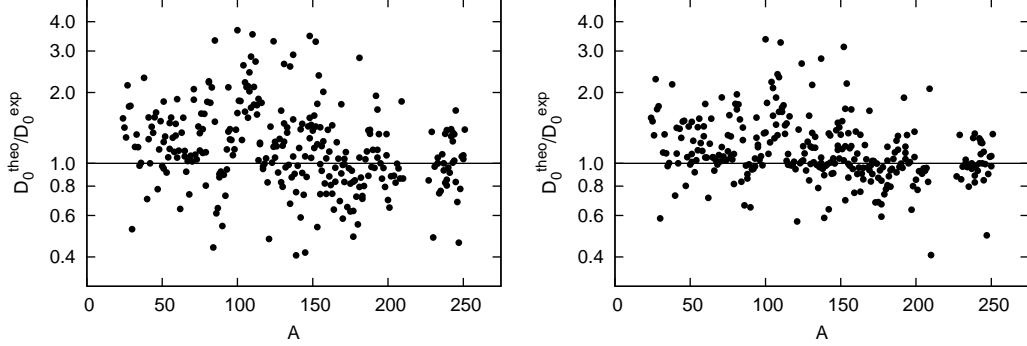


Figure 2: Ratio of the predicted mean resonance spacing to observed value as a function of mass number for 289 stable nuclei. The results of the global and local calculations are illustrated in left and right panels, respectively.

no significance difference between the results of the global and local calculations, except from the general improvement in agreement between experiment and theory. This result is very promising for the reliable extrapolation of the global parameters for the nuclei far from stability.

The collective amplitude  $A_c$  values obtained from the global calculation for 1136 nuclei, which have sufficient information on the discrete energy level scheme, by Eq. (23) are presented in Figure 3. The newly proposed formula (14) includes both vibrational and rotational effects. The moment of inertia, which is given by (18), provides to include the rotational effects into the spin cut-off parameter. The vibrational effects are taken into account via both  $A_c$  and  $E_0$ , with the shape dependent mass formula and the energy of the first phonon state, respectively. Besides the increasing magnitude of the collective amplitude with the mass number, its values are also separated for odd-odd, odd and even-even nuclei as clearly seen from Figure 3. Therefore, it can be concluded that the pairing of the valence nucleons has a strong influence on the collective excitations.

Table 2: The comparison of the predictive power of the phenomenological level density models, including the model presented in this paper. The  $f_{\text{rms}}$  covers 289 nuclei, which naturally exist on earth, and  $f_{\text{lev}}$  covers 1136 nuclei, which have sufficient experimental information on their discrete level scheme.

Model	Type	$f_{\text{rms}}$	$f_{\text{lev}}$
This work (Local)	Collective	1.34	0.98
This work (Global)	Collective	1.53	1.32
Semi-classical BSFGM [26]	Effective	1.12	43.9
BSFGM [20]	Effective	1.68	28.5
BSFGM [20]	Collective	1.71	35.3
CGCM [20]	Effective	1.76	24.2
CGCM [20]	Collective	1.77	47.8
GSM [20]	Effective	1.78	28.0
GSM [20]	Collective	1.94	47.4



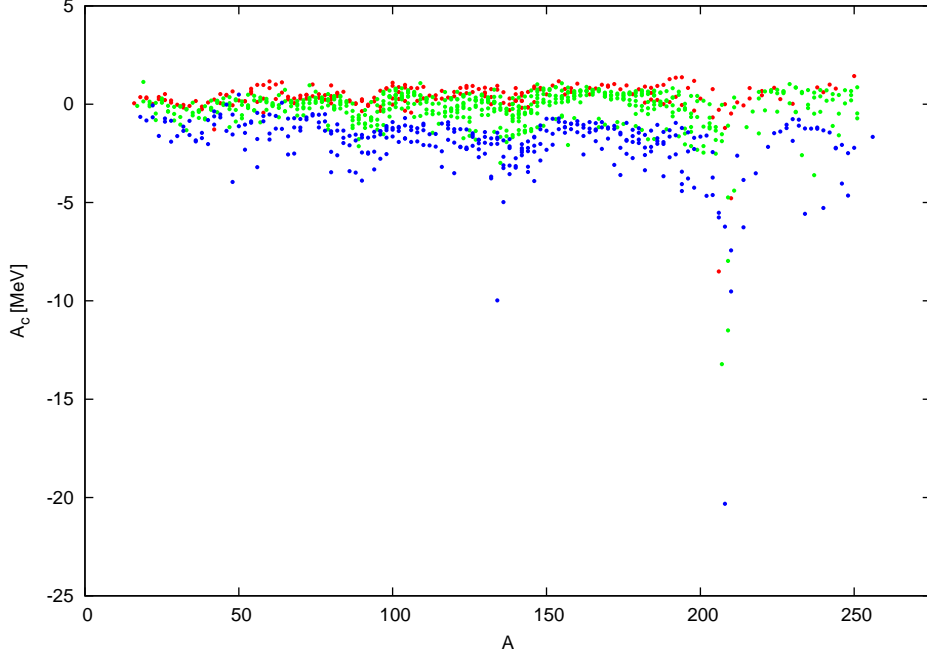


Figure 3: The collective amplitude values obtained from the global calculation for 1136 nuclei. The red, green and blue dots stand for the odd-odd, odd and even-even nuclei respectively.

The goodness-of-fit estimators,  $f_{\text{rms}}$  and  $f_{\text{lev}}$ , values obtained from the both local and global calculations are given in Table 2. All the other models, except from the model presented in this paper, have several adjustable parameters to be determined from the both mean resonance spacings and discrete level schemes. However, even if the reliable information on the discrete levels is available for over 1000 nuclei, the mean resonance spacings data are found only for less than 300 nuclei, which exist naturally, the dependence to these data reduces the applicability and reliability of the models. Because of the lack of experimental data, the studies of the nuclei far from stability are based on an extrapolation. Our model does not include any of these adjustable parameters. It only depends strongly on the shape dependent mass formula, and experimental mass data are available for almost every nucleus. Aside from all these advantages, our model gives the best agreement with the experiments when compared to other models.

The level schemes can be constructed by using the excitation energies that the integral (8) gives discrete integer values when these energies are used as the upper limit. The predicted level schemes for arbitrarily chosen nuclei,  $^{148}\text{La}$ ,  $^{200}\text{Au}$ ,  $^{49}\text{Ti}$ ,  $^{46}\text{V}$ , and also  $^{105}\text{Pd}$ ,  $^{136}\text{Xe}$ , which are known as typical collective nuclei, are shown in Figure 4. All the predicted overlaps and gaps of energy levels are completely in agreement with the observed data. It is also seen from the right panel of Figure 4 that our cumulative level calculations for  $^{49}\text{Ti}$ ,  $^{46}\text{V}$  estimate a first excited state at very low energy, which is absent in the experimental data. Since this state is very close to ground state, maybe it will

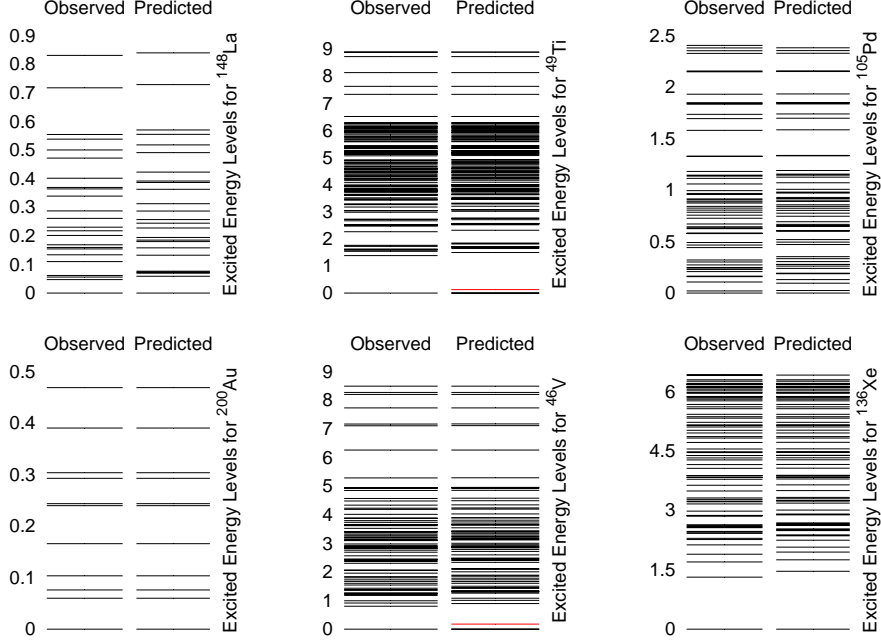


Figure 4: Experimental and predicted level schemes of  $^{148}\text{La}$ ,  $^{200}\text{Au}$ ,  $^{49}\text{Ti}$ ,  $^{46}\text{V}$ ,  $^{105}\text{Pd}$ , and  $^{136}\text{Xe}$ . Predicted level schemes are obtained from the global calculation.

have a facilitating effect on the unresolved quasi-elastic cross-section issue of the light exotic nuclei [36].

#### 4. Conclusions

The analytic calculation of the asymptotic level density parameter including the shell and pairing effects leads the semi-classical approach previously described in Ref. [26] to more physical point. In addition, the slight difference between the results of the global and local fit increases the reliability of the extrapolation of the global parameters to mass region from the stability valley to drip lines.

Another point which must be stressed here concerns the predicted levels of the excited states very close to ground state. Our model estimates this kind of levels, which has not observed experimentally yet, but they may have considerable effect in quasi-elastic cross-section of the light exotic nuclei. Further calculations of the quasi-elastic cross-section, which include these levels as inelastic absorption, will be necessary to understand the presence of these levels.

In summary, a new formula is proposed for the energy dependence of the level density parameter including collective effects. The results obtained by using this new formula provide an evidence that the level density parameter is the correct variable to include

the collective effects and also show great improvement in agreement between observed and predicted energy levels as seen from Table 2.

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